

Finite BRST-BFV transformations for dynamical systems with second-class constraints

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Abstract

We study finite field dependent BRST-BFV transformations for dynamical systems with first- and second-class constraints within the generalized Hamiltonian formalism. We find explicitly their Jacobians and the form of a solution to the compensation equation necessary for generating an arbitrary finite change of gauge-fixing functionals in the path integral.

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1 Introduction

As far as the Hamiltonian constrained dynamics is concerned, it is well-known that one can always convert original second-class constraints into first-class ones by introducing extra degrees of freedom [1, 2, 3, 4, 5, 6, 7]. Thus, in principle, one is always allowed to deal with first-class constraints only. However, because of some specific reasons one can do prefer to work directly with original second-class constraints, as they defined by Dirac [8, 9] (see also models [10, 11]). Here we recall some elementary facts as to the construction of the path integral for the partition function in that case. A new feature in our analysis is that the invariance of the formalism under rotation of second-class constraints is also shown to be a kind of a BRST symmetry in miniature.

2 Pure second-class constraints

Let

$$Z^A = (P_i, Q^i), \quad \varepsilon(P_i) = \varepsilon(Q^i), \quad (2.1)$$

be a set of original canonical variables. Let

$$H(Z), \quad \varepsilon(H) = 0, \quad (2.2)$$

be an original non-degenerate Hamiltonian, and let

$$\Theta^\alpha(Z), \quad \varepsilon(\Theta^\alpha) = \varepsilon_\alpha, \quad (2.3)$$

be original second-class constraints, so that their Poisson bracket matrix,

$$\{\Theta^\alpha, \Theta^\beta\}, \quad (2.4)$$

is invertible. Let us define the action

$$W = \int dt \left[\frac{1}{2} Z^A \omega_{AB} \frac{dZ^B}{dt} - H - \Theta^\alpha \xi_\alpha - \frac{1}{2} C_\alpha \{\Theta^\alpha, \Theta^\beta\} C_\beta \right], \quad (2.5)$$

where ω_{AB} is an inverse to

$$\omega^{AB} = \{Z^A, Z^B\} = \text{const}(Z), \quad (2.6)$$

ξ_α are Lagrange multipliers, $\varepsilon(\xi_\alpha) = \varepsilon_\alpha$, C_α are Dirac ghosts, $\varepsilon(C_\alpha) = \varepsilon_\alpha + 1$.

The partition function is given by the path integral

$$\mathcal{Z} = \int [DZ][D\xi][DC] \exp \left\{ \frac{i}{\hbar} W \right\}. \quad (2.7)$$

The action (2.5) is invariant under the following "BRST transformations" with μ being a fermionic parameter,

$$\delta Z^A = \{Z^A, \Theta^\alpha\} C_\alpha \mu, \quad (2.8)$$

$$\delta C_\beta = \mu \{H, \Theta^\alpha\} D_{\alpha\beta} + \xi_\beta \mu, \quad \delta \xi_\alpha = 0, \quad (2.9)$$

where $D_{\alpha\beta}$ is an inverse to $\{\Theta^\alpha, \Theta^\beta\}$. For constant μ , the Jacobian of the transformations (2.8) and (2.9) equals to one. Thus, the path integral (2.7) is stable under the transformations (2.8) and (2.9) with constant μ .

Now, let us consider a field-dependent Fermionic parameter of the form

$$\mu = \frac{i}{\hbar} \int dt C_\alpha \delta \Lambda_\beta^\alpha \Theta^\beta, \quad (2.10)$$

where arbitrary infinitesimal matrix $\delta \Lambda$ is Z -dependent. In that case, the transformations (2.8) and (2.9) yield the Jacobian,⁴

$$J = 1 - \frac{i}{\hbar} \int dt [\delta \Lambda_\beta^\alpha \Theta^\beta \xi_\alpha + C_\alpha \{\delta \Lambda_\beta^\alpha \Theta^\beta, \Theta^\gamma\} C_\gamma + \delta H], \quad (2.11)$$

which induces arbitrary infinitesimal rotation of the constraints,

$$\Theta \rightarrow (1 + \delta \Lambda) \Theta, \quad (2.12)$$

in the integrand in (2.7), accompanied by the weakly vanishing variation of the Hamiltonian,

$$\delta H = -\{H, \Theta^\alpha\} D_{\alpha\beta} \delta \Lambda_\gamma^\beta \Theta^\gamma \simeq 0. \quad (2.13)$$

Equivalently, one can say that the δH , (2.13), can be compensated by the corresponding shift of ξ_γ to the first order in $\delta \Lambda$,

$$\delta \xi_\gamma = -\{H, \Theta^\alpha\} D_{\alpha\beta} \delta \Lambda_\gamma^\beta (-1)^{\varepsilon_\gamma}. \quad (2.14)$$

Thus, we have confirmed that the path integral (2.7) is, in fact, independent of the special choice of the basis of constraints. On the other hand, by rotating the basis, one can always make the Poisson bracket matrix of the constraints be constant, so that the Dirac ghosts decouple, and the path integral reduces explicitly to the physical degrees of freedom.

⁴In accordance with the general ideology of Ref. [12], the transformations (2.8) and (2.9) can also be generalized to the case of finite field-dependent fermionic parameter μ , although their Jacobians in that case are modified essentially with the terms containing explicitly the differential squared, corresponding to (2.8), (2.9), as applied to the Fermionic BRST parameter μ , $\ln J = -\ln(1 + \kappa) - (\mu \overleftarrow{d}^2)(1 + \kappa)^{-1} \mu$, $\kappa = (\mu \overleftarrow{d})$ (see also [13]), where the differential is defined via the transformations (2.8) and (2.9) as $\overleftarrow{d} = \int dt [(\overleftarrow{\delta} / \delta Z^A) \delta Z^A + (\overleftarrow{\delta} / \delta C_\beta) \delta C_\beta] (\overleftarrow{\partial} / \partial \mu)$. However, as the latter differential is not nilpotent, one cannot guarantee the existence of a solution for a finite fermionic parameter μ generating an arbitrary finite rotation of the constraints (2.3).

Finally, let us rewrite the path integral in its "conceptual" form,

$$\mathcal{Z} = \int \exp \left\{ \frac{i}{\hbar} \int dt \left[\frac{1}{2} Z^A \omega_{AB} \frac{dZ^B}{dt} - H(Z) \right] \right\} d\mu[Z], \quad (2.15)$$

with the functional measure

$$d\mu[Z] = \delta[\Theta] \rho[Z] [DZ], \quad (2.16)$$

where

$$\delta[\Theta] = \int [D\xi] \exp \left\{ -\frac{i}{\hbar} \int dt \Theta \xi \right\} = \prod_t \delta(\Theta), \quad (2.17)$$

is a functional δ - function of the constraints,

$$\begin{aligned} \rho[Z] &= \int [DC] \exp \left\{ -\frac{i}{2\hbar} \int dt C \{ \Theta, \Theta \} C \right\} = \\ &= \exp \left\{ \delta(0) \frac{1}{2} \int dt \ln (\text{sdet} \{ \Theta, \Theta \}) \right\} = \prod_t \sqrt{\text{sdet} \{ \Theta, \Theta \}} \end{aligned} \quad (2.18)$$

is a measure density as represented in terms of the Pfaffian [17] (see also [18, 19]). Notice an important invariance property of the measure (2.16) under the transformations $\delta Z^A = \{Z^A, G\}_{\mathcal{D}}$, generated canonically by the Dirac bracket on the hypersurface $\Theta^\alpha = 0$,

$$(\rho(Z))^{-1} \partial_A (\rho(Z) \omega_{\mathcal{D}}^{AB}(Z)) (-1)^{\varepsilon_A} = 0, \quad (2.19)$$

$$\omega_{\mathcal{D}}^{AB}(Z) = \{Z^A, Z^B\}_{\mathcal{D}}, \quad \rho(Z) = \sqrt{\text{sdet} \{ \Theta, \Theta \}} \quad (2.20)$$

Recall that for any functions F, G , the Dirac bracket is defined in terms of the Poisson brackets as

$$\{F, G\}_{\mathcal{D}} = \{F, G\} - \{F, \Theta^\alpha\} D_{\alpha\beta} \{\Theta^\beta, G\}, \quad (2.21)$$

where $D_{\alpha\beta}$ is an inverse to (2.4). The Dirac bracket satisfies the antisymmetry, Leibnitz rule and Jacobi identity, in the same sense as usual Poisson brackets do. The Dirac brackets also satisfy

$$\{F, \Theta^\alpha\}_{\mathcal{D}} = 0, \quad (2.22)$$

for any F .

Thus, our final statement here is that there exists a similarity between the arbitrariness in rotation of constraints, and genuine gauge invariance.

3 First- and second-class constraints together

For the sake of technical simplicity, in the present section, we consider only irreducible gauge theories, whose first-class constraints are linearly independent, by definition. We begin with describing the general structure of the extended phase space intended specifically to quantize irreducible gauge theories. Here we denote by Z^A the total set of canonical pairs of the extended phase space,

$$Z^A = (z^i; \pi_a, \lambda^a; \bar{\mathcal{P}}_a, C^a; \bar{C}_a, \mathcal{P}^a), \quad (3.1)$$

(i) (z^i) denotes a set of original canonical variables, their Grassmann parities are (ε_i) , their ghost numbers are (0) ; original Hamiltonian $H_0(z)$, first-class constraints $T_a(z)$, and second-class constraints $\Theta_\alpha(z)$ are regular functions of z^i only, their Grassmann parities are $(0, \varepsilon_a, \varepsilon_\alpha)$; all other canonical variables are split explicitly into pairs of canonical momenta and coordinates; among the latter canonical pairs are:

(ii) Dynamically active Lagrange multipliers to first-class constraints and to their gauges,

$$(\pi_A, \lambda^a), \quad (3.2)$$

their Grassmann parities are $(\varepsilon_A, \varepsilon_a)$, their ghost numbers are $(0, 0)$;

(iii) Ghosts,

$$(\bar{\mathcal{P}}_a, C^a), \quad (3.3)$$

their Grassmann parities are $(\varepsilon_a + 1, \varepsilon_a + 1)$, their ghost numbers are $(-1, +1)$;

(iv) Antighosts,

$$(\bar{C}_a, \mathcal{P}^a), \quad (3.4)$$

their Grassmann parities are $(\varepsilon_a + 1, \varepsilon_a + 1)$, their ghost numbers are $(-1, +1)$.

We proceed with the original Dirac bracket form of the classical gauge algebra,

$$\{T_a, T_b\}_{\mathcal{D}} \simeq U_{ab}^c T_c, \quad \{T_a, H_0\}_{\mathcal{D}} \simeq V_a^b T_b, \quad (3.5)$$

where \simeq means weak equality, modulo arbitrary linear combination of second-class constraints Θ^α [8].

Given the classical gauge algebra (3.5), one defines the fermionic BRST-BFV generator Ω and bosonic extended Hamiltonian \mathcal{H} , to satisfy the gauge-algebra generating equations,

$$\{\Omega, \Omega\}_{\mathcal{D}} \simeq 0, \quad \varepsilon(\Omega) = 1, \quad \text{gh}(\Omega) = 1, \quad (3.6)$$

$$\{\Omega, \mathcal{H}\}_{\mathcal{D}} \simeq 0, \quad \varepsilon(\mathcal{H}) = 0, \quad \text{gh}(\mathcal{H}) = 0. \quad (3.7)$$

The existence of a solution [7, 14, 15, 16] to the gauge algebra generating Eqs. (3.6) and (3.7) is guaranteed by the following consequences of the Jacobi identities for the Dirac brackets,

$$\{\{F, F\}_{\mathcal{D}}, F\}_{\mathcal{D}} = 0, \quad \{\{X, F\}_{\mathcal{D}}, F\}_{\mathcal{D}} = \{X, (1/2)\{F, F\}_{\mathcal{D}}\}_{\mathcal{D}}, \quad \varepsilon(F) = 1, \quad \text{any } X. \quad (3.8)$$

One has to seek for a solution to these generating equations in the form of a ghost power series expansions,

$$\Omega \simeq \mathcal{P}^a \pi_a + \left[C^a T_a + \frac{1}{2} (-1)^{\varepsilon_b} C^b C^a U_{ab}^c \bar{\mathcal{P}}_c (-1)^{\varepsilon_c} + \mathcal{O}(CCC\bar{\mathcal{P}}\bar{\mathcal{P}}) \right], \quad (3.9)$$

$$\mathcal{H} \simeq H_0 + C^a V_a^b \bar{\mathcal{P}}_b (-1)^{\varepsilon_b} + \mathcal{O}(CC\bar{\mathcal{P}}\bar{\mathcal{P}}). \quad (3.10)$$

Respectively, to the CC - and C - order, Eqs. (3.6) and (3.7) reproduce the gauge algebra relations (3.5). Higher structure relations of the gauge algebra are reproduced to higher orders in ghosts.

Define the complete unitarizing Hamiltonian H_Ψ by the formula

$$H_\Psi \simeq \mathcal{H} + \{\Omega, \Psi\}_{\mathcal{D}}, \quad \varepsilon(\Psi) = 1, \quad \text{gh}(\Psi) = -1. \quad (3.11)$$

where Ψ is a gauge-fixing Fermion function of the form

$$\Psi \simeq \lambda^a \bar{\mathcal{P}}_a + \chi^a \bar{C}_a, \quad (3.12)$$

with χ^a being just the gauge functions by themselves. They are allowed to depend on all the phase variables, under the only condition that

$$\text{gh}(\chi^a) = 0. \quad (3.13)$$

Due to (3.6) and (3.7),

$$\{H_\Psi, \Omega\}_{\mathcal{D}} \simeq 0. \quad (3.14)$$

To the second order in ghosts, with $\chi^a = \chi^a(z; \pi, \lambda)$, the unitarizing Hamiltonian is

$$\begin{aligned} H_\Psi &= H_0 + (T_a + (-1)^{\varepsilon_a} C^b U_{ba}^c \bar{\mathcal{P}}_c (-1)^{\varepsilon_c}) \lambda^a + \pi_a \chi^a + \\ &\quad + \bar{C}_a \{\chi^a, T_b\}_{\mathcal{D}} C^b + \bar{C}_a \{\chi^a, \pi_b\}_{\mathcal{D}} \mathcal{P}^b + (C^a V_a^b - \mathcal{P}^b) \bar{\mathcal{P}}_b (-1)^{\varepsilon_b}. \end{aligned} \quad (3.15)$$

Now, define the complete action,

$$W_\Psi = \int dt \left[\frac{1}{2} Z^A \omega_{AB} \frac{dZ^B}{dt} - H_\Psi \right], \quad (3.16)$$

in terms of the unitarizing Hamiltonian (3.11). Then, we define the corresponding path integral [14],

$$\mathcal{Z} = \mathcal{Z}_\Psi = \int \exp \left\{ \frac{i}{\hbar} W_\Psi \right\} d\mu[Z], \quad (3.17)$$

with the functional measure

$$d\mu[Z] = \delta[\Theta]\rho[z][DZ], \quad (3.18)$$

where

$$\delta[\Theta] = \int [D\xi] \exp \left\{ -\frac{i}{\hbar} \int dt \Theta^\alpha(z) \xi_\alpha \right\} = \prod_t \delta(\Theta(z)), \quad (3.19)$$

$$\begin{aligned} \rho[z] &= \int [DC] \exp \left\{ -\frac{i}{\hbar} \int dt \frac{1}{2} C^\alpha \{ \Theta^\alpha(z), \Theta^\beta(z) \} C^\beta \right\} = \\ &= \exp \left\{ \delta(0) \int dt \ln \rho(z) \right\} = \prod_t \rho(z), \quad \rho(z) = \sqrt{\text{sdet} \{ \Theta(z), \Theta(z) \}}. \end{aligned} \quad (3.20)$$

In analogy with (2.19), one has in the sector of the original variables z^i , [14],

$$(\rho(z))^{-1} \partial_i (\rho(z) \omega_{\mathcal{D}}^{ij}(z)) (-1)^{\varepsilon_i} = 0. \quad (3.21)$$

In the path integral (3.17), consider the infinitesimal BRST-BFV transformation,

$$\delta Z^A \simeq \{Z^A, \Omega\}_{\mathcal{D}} \mu. \quad (3.22)$$

On the constraint surface

$$\Theta^\alpha(z) = 0, \quad (\text{any } t), \quad (3.23)$$

the induced variation in the action (3.16) is given by the boundary term,

$$\delta W_\Psi = \left[\frac{1}{2} (Z^A P_A^B \partial_B - 2) \Omega \mu \right] \Big|_{-\infty}^{+\infty}, \quad (3.24)$$

where we have denoted the Dirac projector matrix

$$P_A^B = \omega_{AC} \omega_{\mathcal{D}}^{CB}. \quad (3.25)$$

As to the Jacobian of the transformation (3.22), we have

$$J_{\mathcal{D}} = 1 + \int dt \left[-\mu \overleftarrow{d}_{\mathcal{D}} + \delta(0) (-1)^{\varepsilon_A} (\partial_A \omega_{\mathcal{D}}^{AB}) (\partial_B \Omega) \mu \right], \quad (3.26)$$

where

$$\overleftarrow{d}_{\mathcal{D}} = \int dt \frac{\overleftarrow{\delta}}{\delta Z^A} \{Z^A, \Omega\}_{\mathcal{D}}, \quad (3.27)$$

is the Dirac version of the BRST-BFV differential,

$$(\overleftarrow{d}_{\mathcal{D}})^2 \simeq 0, \quad \varepsilon(\overleftarrow{d}_{\mathcal{D}}) = 1, \quad \text{gh}(\overleftarrow{d}_{\mathcal{D}}) = 1. \quad (3.28)$$

Here and below in operator-valued weak equalities, we mean "normal - ordered" linear combinations of second-class constraints, with functional derivative operators applying to the left, standing to the left of all the rest factors, in every monomial. Due to the relations (2.19) or (3.21), the second term in the square bracket in the right-hand side in (3.26), is compensated exactly by the induced BRST-BFV variation in the density ρ .

It follows then from (3.24) and (3.26) that the path integral (3.17) is stable under the transformations (3.22) with $\mu = \text{const}$, in case of appropriate boundary condition imposed for integration trajectories. On the other hand, if one chooses μ in the form

$$\mu = \frac{i}{\hbar} \int dt \delta \Psi, \quad (3.29)$$

then the Jacobian (3.26) yields effective change of the gauge fermion,

$$\Psi \rightarrow \Psi + \delta \Psi. \quad (3.30)$$

Thereby one has confirmed the formal gauge independence as to the path integral (3.17).

4 Finite BRST-BFV transformations, their Jacobians and compensation equation

Here, we proceed with the finite BRST- BFV transformations in their Dirac-bracket version,

$$\bar{Z}^A \simeq Z^A + \{Z^A, \Omega\}_{\mathcal{D}} \mu = Z^A (1 + \overleftarrow{d}_{\mathcal{D}} \mu), \quad (4.1)$$

$$[\overleftarrow{d}_{\mathcal{D}} \mu_1, \overleftarrow{d}_{\mathcal{D}} \mu_2] \simeq \overleftarrow{d}_{\mathcal{D}} \mu_{[12]}, \quad \mu_{[12]} \simeq -(\mu_1 \mu_2) \overleftarrow{d}_{\mathcal{D}}. \quad (4.2)$$

By exactly the same reasoning as in Ref. [12], see (2.19) and (2.20) therein, it follows that the Jacobian of the transformation (4.1) has the general form

$$\ln J_{\mathcal{D}} \simeq -\ln(1 + \kappa_{\mathcal{D}}) + \delta(0) \int dt (-1)^{\varepsilon_A} (\partial_A \omega_{\mathcal{D}}^{AB}) (\partial_B \Omega) \mu, \quad (4.3)$$

where

$$\kappa_{\mathcal{D}} = \mu \int dt \frac{\overleftarrow{\delta}}{\delta Z^A} \{Z^A, \Omega\}_{\mathcal{D}} = \mu \overleftarrow{d}_{\mathcal{D}}. \quad (4.4)$$

To the first order in μ , (i.e. in the infinitesimal case) (4.3) does coincide with (3.26). The same as in the latter case, the second term in the right-hand side in (4.3) is compensated, due to (2.19) or (3.21), by the BRST-BFV variation of the density ρ in the functional measure $d\mu[Z]$. Thus, the first term in the right-hand side in (4.3), is the only contribution to formulate the compensation equation as to the path integral (3.17),

$$\overleftarrow{\mu d}_{\mathcal{D}} \simeq \exp \left\{ \frac{i}{\hbar} \left(\delta \Psi[Z] \overleftarrow{d}_{\mathcal{D}} \right) \right\} - 1, \quad \Psi[Z] = \int dt \Psi(Z(t)). \quad (4.5)$$

An obvious solution to that equation has the form

$$\mu[\delta\Psi] = \frac{i}{\hbar} E((i/\hbar)(\delta\Psi[Z]\overleftarrow{d}_{\mathcal{D}}))\delta\Psi[Z], \quad E(x) = x^{-1}(\exp x - 1). \quad (4.6)$$

If one chooses the variables (4.1) with parameters (4.6), to be the new integration variables in the path integral (3.17), then, in the new variables, one gets the new gauge fermion, $\Psi_1 = \Psi + \delta\Psi$. If one introduce external source $J_A(t)$, to define the generating functional,

$$\mathcal{Z}_\Psi = \int d\mu[Z] \exp \left\{ \frac{i}{\hbar} \left[W_\Psi + \int dt J_A Z^A \right] \right\}, \quad (4.7)$$

then the following interpolation formula between the two finite-differing gauges, Ψ_1 and Ψ , holds

$$\mathcal{Z}_{\Psi_1} = \mathcal{Z}_\Psi \left[1 + \left\langle \frac{i}{\hbar} \int dt J_A \left(Z^A \overleftarrow{d}_{\mathcal{D}} \right) \mu[-\delta\Psi] \right\rangle_\Psi \right], \quad (4.8)$$

where $\mu[\delta\Psi]$ is given by (4.6), and the quantum mean value, $\langle(\dots)\rangle_\Psi$, is defined by

$$\langle(\dots)\rangle_\Psi = (\mathcal{Z}_\Psi)^{-1} \int d\mu[Z] (\dots) \exp \left\{ \frac{i}{\hbar} \left[W_\Psi + \int dt J_A Z^A \right] \right\}. \quad (4.9)$$

Thus, one has confirmed that finite BRST-BFV transformations in their Dirac-bracket version are quite capable of inducing finite change of gauge-fixing fermion in the path integral in the presence of second-class constraints. As the situation with the other aspects of the matter is quite obvious, we have no reason to consider the aspects here in further detail (see [12]).

5 Discussion

In the present article, we have extended our study [12] of finite field-dependent BRST-BFV transformations within the generalized Hamiltonian formalism [20, 21], to the case of the second-class constraints present. It was shown that the invariance of the formalism under rotations of second-class constraints can be represented in the form of a BRST-like symmetry. An explicit form of the Jacobian of the finite BRST-BFV transformation was found in terms of the Dirac-bracket version of the functional differential applying on the space of trajectories. We have formulated the compensation equation determining the finite parameter of the BRST-BFV transformation to make its Jacobian yield arbitrary finite change in the gauge-fixing fermion function. It was confirmed that all the results of [12] generalize naturally via replacement of: (i), the ordinary Poisson bracket by the Dirac bracket, (ii) the trivial canonical integration measure in the path integral by the Dirac measure, and (iii) considering all basic equations in the weak sense of Dirac.

In conclusion, we demonstrate how the "conceptual" form (2.15) - (2.18) of the path integral with second-class constraints does generalize as to the case of the general coordinates Z^A , whereas the basic invertible symplectic metric,

$$\omega^{AB}(Z) = \{Z^A, Z^B\}, \quad (5.1)$$

is not a constant in Z . The latter metric, in its contravariant components, does satisfy the Jacobi identity,

$$\omega^{AD}\partial_D\omega^{BC}(-1)^{\varepsilon_A\varepsilon_C} + \text{cyclic perm.}(A, B, C) = 0, \quad (5.2)$$

or, in its covariant components ω_{AB} , $\omega^{AB}\omega_{BC} = \delta_C^A$, one has,

$$\partial_C\omega_{AB}(-1)^{(\varepsilon_C+1)\varepsilon_B} + \text{cyclic perm.}(A, B, C) = 0. \quad (5.3)$$

Then, one should make the following replacements in (2.15) and (2.16). In the integrand in (2.18), in the exponential, in the square brackets, one should replace

$$\frac{1}{2}\omega_{AB} \rightarrow \bar{\omega}_{AB} = (Z^C\partial_C + 2)^{-1}\omega_{AB}, \quad (5.4)$$

in the kinetic part of the action. In (2.16), one should replace

$$\rho[Z] \rightarrow \bar{\rho}[Z] = \exp \left\{ \delta(0) \int dt \ln (\bar{\rho}(Z)) \right\} = \prod_t \bar{\rho}(Z), \quad (5.5)$$

where the new local density is given by

$$\bar{\rho}(Z) = \rho(Z) \sqrt{\text{sdet}(\omega_{AB})}, \quad (5.6)$$

via the "old" local density, the second in (2.20). In the canonical invariance property (2.19), one should replace

$$\rho(Z) \rightarrow \bar{\rho}(Z). \quad (5.7)$$

As to the general-coordinate version of the path integral (3.17), with the first-class constraints present, the latter generalization, in principle, includes the same two steps: one should modify the kinetic part of the action (3.16) via (5.4), and the integration measure (3.18) - (3.20) via (5.5) and (5.6).

It should be also mentioned here that the general-coordinate version of the constrained dynamics generalizes further to the level of a superfield [22, 23], $Z^A(t, \tau) = Z_0^A(t) + \tau Z_1^A(t)$, $\varepsilon(\tau) = 1$, with the covariant derivative $D = (d/d\tau) + \tau(d/dt)$, $D^2 = (d/dt)$, and original action of the form $W = \int dt d\tau [Z^A \bar{\omega}_{AB} D Z^B (-1)^{\varepsilon_B} - Q_\Psi]$ and $[Q_\Psi, Q_\Psi] = 0$. Although we do not go into detail here, note that finite BRST- BfV transformations do correspond, within the superfield formalism, to finite supertranslations along the τ direction. In the superfield path integral, the superfield delta-functional of second-class constraints is included with trivial (constant) measure density; nontrivial density in the original phase space is generated automatically when getting back to a component formalism.

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